IDENTIFYING NEEDED TANGIBLE MATHEMATICAL AIDS THROUGH TEXTBOOK ANALYSIS

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The creation of tangible apparatus to aid in the education of the blind has been a concern for many centuries. Numerous inventions have appeared with mention of the previous discoveries leading to their development, but recorded evidence about such aids is extremely rare. This is particularly true in regard to evidence which is substantiated empirically. Teachers in the field of education of the blind are aware of the kinds of tangible aids needed. They are, of course, not researchers and their observations as to the efficiency of educational aids are individual observations. The lack of empirical evidence to substantiate these observations accounts, in part, for the fact that many needed tangible aids are not available at the present time.

A survey of the literature in this field reveals that there has been no consistent pattern in the development of tangible aids for visually handicapped children. Various aids have simply been developed as the need for them arose and, many times, the available information regarding problems in tactual perception and tactual discrimination was not considered. As a result, the aids developed and identified in this way met only the specific needs for those for whom they were developed. The realization of this lack of consistency revealed the need for a technique which would use knowledge gained from previous research, combined with a systematic analysis of the educational materials to be used by the visually handicapped child, to

develop tangible aids which would meet all his needs.

This article is based on a study entitled Textbook Analysis As A Technique for Identifying Tangible Mathematical Aids for Visually Handicapped Children (Caton, 1969) which was conducted at the American Printing House for the Blind. In the study, the technique of textbook analysis was utilized and, as a result, the Geometric Area and Volume Aid (GAVA) was developed. The results of the study indicated clearly that it is possible to use textbook analysis as a basis for the identification and development of tangible aids for visually handicapped children.

Textbook Analysis

Textbook analysis, in this study, involved a careful survey of the existing literature on tangible apparatus, as well as a comparison and analysis of illustrative material in both braille and inkprint editions of the Addison-Wesley Elementary School Mathematics Series. The analysis was conducted in order that tangible aids could be identified which would be specifically designed to supplement or replace illustrative materials contained in the inkprint editions of any elementary mathematics textbook series.

The procedure used to demonstrate the effectiveness of the technique of textbook analysis was as follows:

1. An extensive survey of existing literature was conducted in order to determine the present status of research in this area, and to study methods of identifying aids which were used in the past.
2. A comparison of the illustrative materials in the braille and inkprint editions of the textbooks was conducted in order to determine which illustrations had been omitted from the braille textbooks, and which had been inadequately reproduced in the braille texts as raised-line illustrations.
3. A classification of the omitted and inadequately reproduced

material was made. The materials were classified as essential or non-essential, in ~~the~~ terms of the actual need for the illustration in solving the problems.

4. Frequency tables were constructed in order to determine which illustrations appeared most frequently so that these might receive first priority when tangible aids were developed or identified. lists
5. A search of current commercial catalogs was made and tangible aids which were available and those not available commercially was made.
6. A set of geometric blocks (GAVA) was developed which was designed to supplement the illustrations related to finding volume and surface area. The need for these blocks was determined by studying the frequency tables to determine the areas of greatest need, and by checking the lists of aids not available commercially to determine whether or not they had been developed previously.
7. A pilot demonstration, using the volume blocks, was conducted to determine whether or not they would actually be of value to visually handicapped children. The results indicated that the performance of students using the blocks was superior to their performance when they did not use them.

The construction of the frequency tables indicated very clearly that the illustrative materials fell into definite categories and, therefore, tangible aids could be developed to supplement or replace whole groups of illustrations. This would eliminate the tedious and time consuming process of searching for tangible aids for each illustration as it is encountered in the textbook.

<u>The basic approach</u> used in the study was to gear the investigation to a detailed analysis of the illustrative materials contained in mathematics texts. It was discovered that inkprint textbooks contain ever increasing amounts of illustrative materials and, further, that there has been little systematic investigation to determine how effectively reproductions and adaptations of these materials are meeting the needs of visually handicapped children. Therefore, the basic question, which had to be answered in relation to solving this problem was "How can the <u>information</u> involved in the illustrations in braille textbooks be conveyed to the tactual reader?" (Nolan, 1964).

Development of the Geometric Area and Volume Aid (GAVA)

With the above view in mind, investigations were pursued which pinpointed more clearly the area of highest probable difficulty. The illustrations in this category appeared to be those which were three-dimensional or which required that the student have some concept of space, depth, height, or perspective. A study of the frequency tables indicated that there was a definite need for some means of constructing models of the three-dimensional figures relating to the concepts of volume and surface area, since it was extremely difficult for visually handicapped students to read raised-line illustrations of three-dimensional figures.

The Geometric Area and Volume Aid (GAVA) was developed as the result of the identification of the need for three-dimensional models. The first step in the development of this aid was to search the current commercial catalogs for materials which might be suitable, and commercially available, for this purpose. The materials needed were sets of 1-inch square blocks, some of which were cut on the diagonal, which could be connected in some way to form the desired figures and then disconnected, at will, to form other figures. There were ~~not~~ no sets of blocks of this type available commercially. Therefore, the Geometric Area and Volume Aid, which had the desired characteristics, was designed.

The specifications for the size and shape of the blocks were determined by examining the textbooks which contained lessons on finding volume and surface area. It was necessary to have materials which could be used to construct exact reproductions of the figures which were illustrated in the textbooks. This accounted for the need for blocks which were cut on the diagonal and for the size requirement of 1-inch square.

The GAVA set ~~is~~ was constructed so that it is possible to connect and disconnect the blocks in such a way that a number of different figures can be constructed, since any one figure can be disassembled and another figure constructed from the same blocks.

<u>The completed GAVA set</u> of geometric blocks (figure 1) for finding volume and surface area consists of 30 unpainted, wooden blocks (10 different shapes), 20 connector pegs, and a wooden storage/carrying tray. With these blocks it is possible to construct models of three-dimensional figures relating to surface area and volume which are most often found in elementary mathematics texts.

The unit size of each block is one cubic inch. The set consists of the wooden tray divided into 11 sections, 6 single-unit blocks, plus 24 multiple-unit blocks, and the 20 connector pegs. The multiple-unit blocks are made tactile by incised grooves which provide for easy unit discrimination. Each of the different blocks is assigned a code-letter identifier (figure 2) and is stored in the section of the wooden tray which carries the corresponding letter. The code-letter identifiers made it possible to construct various figures according to construction formulas (figure 3). Construction of the figures is accomplished by inserting the connector pegs into the blocks to hold them together.

<u>Using the GAVA set in finding volume and surface area</u>

The student determines volume by counting the number of cubes in each figure constructed with the blocks. He finds surface area by counting the number of sides he can touch on each figure. Some suggested activities for finding volume and surface area are as follows:

Instructions - Volume

You can find the volume of each figure by counting the number of cubes it contains. We call each cube a "cube unit". You will give your answer in "cube units" not in inches. You must only give the number of cubes in each figure. You must always remember to count the cubes you cannot touch when the figure is sitting on the table.

For Example:

1. The volume of this figure is 6 units.

   (figure 4)

2. The volume of this figure is 9 units.

   (figure 5)

3. The volume of this figure is 12 units.

   (figure 6)

Instructions - Surface Area

You can find surface area by counting only the sides you can touch. Remember that each cube is called a "cube unit" and that you will give your answer in "cube units"

For Example:

1. You can touch 5 sides of this cube when it sits on the table. When you hold the cube in your hand you can touch 6 sides. Therefore, the surface area of the cube is 6 units.

   (figure 7) connected

2. When two cubes are ~~glued together~~ to make a figure like this, how many sides can you touch? Remember to count only the sides you can touch! You can touch ten sides in this figure. Therefore, the surface area is 10 units.

   (figure 8) connected

3. When three cubes are ~~glued together~~ to make a figure like this, how many sides can you touch? The answer is fourteen, therefore, the surface area is 14 units.

   (figure 9)

Pilot Demonstration of the Geometric Area and Volume Aid

In order to demonstrate the effectiveness of the Geometric Area and Volume Aid, a pilot demonstration was conducted, using this aid, with the

assistance of students from the Kentucky School for the Blind, Louisville, Kentucky. The Gava Set was not in its completed form at that time. It is assumed that the results of the pilot demonstration would have been more positive if it had been used in its completed form.

The criteria for the selection of participating students was based upon the analysis of the textbooks which was described earlier. The textbook examination revealed that the introductory material for the concepts of volume and surface area was contained primarily in the third grade textbook. The bulk of the new material illustrating these concepts was presented in the fourth grade texts, while the fifth and sixth grade books contained material which was basically a review of these concepts. For this reason, the students for the demonstration were chosen on the basis of their grade level as determined by their arithmetic achievement scores and I.Q., rather than by their chronological age or actual school placement. It was felt that children with I.Q.'s in the normal range and arithmetic achievement scores in the third and fourth grade levels in arithmetic achievement would be suitable for the demonstration, since these were the grade levels in which the bulk of the material was presented. The children were all braille readers, since the aid was designed for use by children with little or no vision.

The first step in the pilot demonstration consisted of a pretest in which the student was asked to find the volume and surface area of various figures presented to him by the instructor. No instructions were given regarding methods for doing this, except for an explanation that the 1-inch square blocks were to be used as a unit of measure. The answers were to be given in terms of "cube units" and not in terms of actual inches. The students were required to use, first, raised-line illustrations, and then,

figures constructed from the set of blocks already described. The pretest was given for two reasons. One, to determine, as nearly as possible, what information the students had regarding volume and surface area and, two, to evaluate the difference in their ability to read raised-line illustrations and to use the figures constructed from the blocks.

The results of the pretest indicated that the students had some knowledge concerning volume, but practically no knowledge concerning surface area. All students had completed the fourth grade in which these concepts were taught. Therefore, the lack of knowledge was probably caused by difficulty in the understanding of the figures involved. The students had a great deal of difficulty in reading the raised-line drawings correctly and were not able to distinguish the figures concerned with surface area at all. This seemed to be due to the fact that they were required to distinguish the sides of the raised-line illustrations when finding surface area.

<u>The next step in the pilot demonstration</u> was to instruct the students in the use of the blocks and the reading of raised-line illustrations when finding volume and surface area. Following the instruction, the students were retested on the concepts of volume and surface area. They were given the same figures to identify which were used on the pretest, but they were not informed of this fact.

The results indicated that there was some improvement in the scores after instruction using both the blocks and the illustrations. The improvement might have been partially due to the fact that students were somewhat familiar with the figures, although they gave no indication that they recognized them.

The final step in the pilot demonstration was to administer a posttest which was composed of 20 entirely new models and illustrations. The number of items in the posttest was increased in order to give the students more opportunity to demonstrate their knowledge of the concepts tested.

The results of the pilot demonstration indicated that the student's performance when using the models of figures constructed from the Geometric Area and Volume Aid was definitely superior to their performance when using raised-line illustrations of the same figures. The scores on surface area tests were consistently lower than those on volume tests. The discrepancy in these areas probably occured because the whole blocks used in finding volume were much more ~~easily distinguished~~ distinguishable tactually than the sides of the blocks, which were used to find surface area. However, the scores on surface area were slightly higher when the students used models constructed from blocks, than when raised-line illustrations were used.

The scores on all the tests in which the students used raised-line illustrations were very low. The students' reactions to the drawings were extremely negative. They had a great deal of difficulty in locating the sides and tops of the three-dimensional drawings and in distinguishing one block from another in the same drawing. They also experienced difficulty in reading raised-line illustrations which required knowledge of depth perception or perspective.

The students' reactions to the Geometric Area and Volume Aid were positive and enthusiastic. The fact that their scores on all the tests in the pilot demonstration were higher when the set was used indicates that tangible aids such as this could be of value to visually handicapped students.

Field testing of the Geometric Area and Volume Aid was conducted after the completion of the study in order to evaluate its effectiveness when used by visually handicapped children. The results of this testing indicated that this was a valuable aid to children who are learning the concepts of volume and surface area.

Available from the American Printing House for the Blind

The Geometric Area and Volume Aid (GAVA) is now being produced by the American Printing House for the Blind and can be purchased on Federal Quota accounts.

Summary

The study discussed in this article was designed to demonstrate the technique of textbook analysis as a means of identifying tangible mathematical aids needed by visually handicapped children. This technique is a research-oriented approach to the problem of providing tangible aids to visually handicapped students. In the study, it involved a careful survey of the existing literature, as well as a comparison and analysis of illustrative material in both braille and inkprint editions of the Addison-Wesley Elementary School Mathematics Series (Eicholz, et al, 1964). The analysis was conducted in order that tangible aids could be identified which could be specifically designed to supplement or replace illustrative materials contained in any elementary mathematics series. Such an analysis provides a more systematic and scientific method of identifying tangible aids than methods previously used.

The Geometric Area and Volume Aid was developed to supplement the illustrations concerning volume and surface area. The need for this aid

was determined by studying the frequency tables to identify the areas of greatest need, and by checking the lists of aids not available commercially to see whether or not such an aid had been developed previously. A pilot demonstration and field testing of the Geometric Area and Volume Aid furnished proof that it was, indeed, effective when used by visually handicapped children in finding volume and surface area. For this reason, it is now ~~not~~ being produced by the American Printing House for the Blind and can be purchased on Federal Quota Accounts.

The results of the study indicate clearly that it is possible to use the technique of textbook analysis in the identification and development of tangible aids for visually handicapped children. Using this technique, it is possible to provide lists of tangible aids which are available commercially, and to provide names and addresses of firms where they can be purchased. It is also possible to provide lists of aids which are needed, but are not available commerically and which should be developed. Therefore, it appears that this technique of textbook analysis provides a valuable, research-based approach to the identification of tangible aids which appears to be superior to other less systematic approaches which have been used in the past.

References

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